Inflection point inflation and time dependent potentials in string theory

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ABSTRACT: We consider models of inflection point inflation. The main drawback of such models is that they suffer from the overshoot problem. Namely the initial condition should be fine tuned to be near the inflection point for the universe to inflate. We show that stringy realizations of inflection point inflation are common and offer a natural resolution to the overshoot problem.

Contents

1. Introduction

Recent experimental data [1] provides evidence for an early universe inflation [2 – 4] and quite remarkably even makes predictions about some of the parameters that characterize models of inflation.

String theory, however, does not seem to provide, as yet, a particularly natural setup for inflation (for reviews of recent progress see [5, 6]). The basic problem is that the typical potentials for the scalars in string theory are simply not flat enough to generate a period inflation.

Recently [7 – 11] it was realized that there is a relatively simple way to evade this problem. If the inflaton potential has an inflection point (or an almost inflection region) then a large amount of inflation will be generated around the inflection point (region), provided that the inflaton spends enough time in this region.[1] Thus if the initial condition for the inflaton is near the inflection point, then the number of e-foldings will be large (in fact, very large). However, if the initial condition of the inflaton is away from the inflection point (which is the generic case), then the inflaton will overshoot the inflection point without inflating the universe. In other words, the Hubble friction is not sufficient to slow down the inflaton at the inflection point, and the inflaton will not spend enough time near the inflection point to generate inflation. This can be viewed as a limit of the overshoot problem discussed in [15].

The aim of this paper is twofold. First, we demonstrate in the context of modular inflation [16, 17] that there are a lot of simple examples of inflection point potentials in string theory.[2] Second, we show that quite generically string theory resolves the main drawback of inflection point inflation - the overshoot problem discussed above. The nice aspect of this resolution is that it involves stringy degrees of freedom and so the supergravity

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[1]Inflection point inflation was discussed also in the context of MSSM [12 – 14].

[2]More complicated examples, in the context of brane inflation were discussed recently in [8, 9, 11, 10].

fields by themselves are not sufficient (for other examples of stringy degrees of freedom that help in stabilization of moduli fields, namely through trapping by particle production near ESPs, see [18 – 21]).

The paper is organized as follows. In section 2 we summarize some of the general features of inflection point inflation that are relevant for our work. In section 3 we review the different terms in the potential for the radion and show that an inflection point potential can be easily constructed using these terms. We start section 4 by numerically demonstrating the overshoot problem associated with inflection point inflation. Then we describe the stringy mechanism that resolves this problem. We conclude with some comments in section 5.

2. Inflection point inflation

In this section we collect some of the general properties of inflection point inflation (IPI). Most of the observations made in this section can be found in [12 – 14].

We assume that the inflaton, denoted by ϕ, has a canonically normalized kinetic term and a potential with an inflection point at $\phi_{\text{inflection}}$. Therefore the potential near $\phi_{\text{inflection}}$ takes the form

$$V(\phi) = V_{\text{inflection}} - \beta(\phi - \phi_{\text{inflection}})^3, \tag{2.1}$$

and the slow roll parameters are

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{9\beta^2(\phi - \phi_{\text{inflection}})^4}{2V_{\text{inflection}}^2}, \qquad \eta = \frac{V''}{V} = -\frac{6\beta(\phi - \phi_{\text{inflection}})}{V_{\text{inflection}}}. \tag{2.2}$$

We see that near $\phi_{\text{inflection}}$ the conditions for slow roll inflation are satisfied, and that $\epsilon \ll \eta$. This will play a role momentarily.

Theoretically the nicest feature of IPI is that during inflation the expectation value of ϕ need not vary over super-Planckian distances as can be seen from

$$N = \int \frac{V}{V'}d\phi \approx \frac{V_{\text{inflection}}}{3\beta(\phi_{\text{start}} - \phi_{\text{inflection}})}. \tag{2.3}$$

This is of particular importance in string theory where the inflaton typically has either a direct or indirect (U-dual) geometrical meaning, which makes it problematic to vary it over super-Planckian distances.

At a more practical level the big advantage of IPI is that if the low energy approximation one uses is valid at $\phi_{\text{inflection}}$ then it is valid throughout the period of inflation. In the context of string theory this means that if the supergravity approximation is valid at $\phi_{\text{inflection}}$ then it can be used to describe the whole process of inflation (but not necessarily of re-heating). This is in contrast with other models of inflation where, typically, the need to generate a large number of e-foldings pushes the inflaton either away from the region of validity of the approximation or away from the region of slow roll.

As far as observation goes the sharpest prediction of IPI is that the spectral index, n_s, is smaller than 1 by a considerable amount. This is obtained from the following consideration.

In the slow roll approximation the spectral index is given by

$$n_s \approx 1 - 6\epsilon + 2\eta \approx 1 + 2\eta \approx 1 - \frac{12\beta(\phi_{\text{start}} - \phi_{\text{inflection}})}{V_{\text{inflection}}}, \tag{2.4}$$

where we have used the fact that $\epsilon \ll \eta$. Combining this with (2.3) we find that[3]

$$n_s \approx 1 - \frac{4}{N} \approx 0.933, \tag{2.5}$$

where the last estimate is for $N = 60$. This value of n_s is within the range of current observational limits that give (for small r) $n_s = 0.95 \pm 0.02$.

It would have been great if eq. (2.5) was a sharp prediction of IPI. This, however, is not the case since there are at least two kinds of corrections to eq. (2.5) that are not negligible. The first is due to the fact that a more generic situation is to have an approximate inflection point than an exact inflection point. This will clearly modify the prediction for n_s. The second modification is due to time dependent potentials that appear in string theory and will play a key role in the present paper. Thus eq. (2.5) should not be viewed as an exact prediction of the models discussed here. However, since both modifications are expected to be small, the fact that n_s is smaller than 1 by a significant amount is a prediction of the models of (almost) IPI.

As mentioned above a key feature of IPI is that the universe inflates only when $\phi \sim \phi_{\text{inflection}}$. This implies that models of IPI are highly sensitive to the initial condition. If the initial condition is such that ϕ_{initial} is near $\phi_{\text{inflection}}$ then the model works fine, in the sense that N is large. But for generic values of ϕ_{initial} the inflaton acquires a large velocity by the time it reaches $\phi_{\text{inflection}}$ and it simply crosses the inflection point without inflating the universe. Simply put, the inflaton overshoots the inflection point. It is the purpose of this paper to show that stringy realizations of IPI resolve this problem.

For later use we also mention that the COBE normalization condition gives

$$\frac{V_0}{\beta^2} \approx 0.33 \ 10^8 N^4. \tag{2.6}$$

In the next section we shall see that it is easy to satisfy this condition in the stringy models of inflection point inflation.

3. IPI in string theory

Our goal in this section is to show that models of IPI can be found in string theory. In the context of brane inflation this was shown in [8 – 10]. Here we focus on modular inflation in which one of the moduli fields is the inflaton and the other moduli fields are assumed to be stabilized with a higher mass than the characteristic mass scale associated with the inflaton (for a recent review on moduli stabilization see [22]). We consider the case where the inflaton is the radion. We start by summarizing the various known contributions to the potential of the radion, and then we show that they can be combined to yield IPI.

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[3]Here we are assuming that the inflanton does not overshoot the inflection point. As we shall see this is the case in the stringy realization of IPI.

The setup we work with is the usual one with a compact manifold M of dimension $d = 6$. For simplicity we assume that the compact manifold is characterized by one length scale, L, and so the volume of M is $\text{Vol}_M = cL^d$, where c is a dimensionless constant of order one.

Before we discuss the various ways to generate a potential for L we recall that it is useful to describe the potential in the Einstein frame, where the kinetic term for L does not mix with the kinetic term of the graviton. This ensures that the potential we find for L can be interpreted in the standard way. We start with the Einstein-Hilbert action in $s = 4 + d$ dimensions and KK reduce it to four dimensions to find

$$\frac{1}{16\pi G_s} \int d^s x \sqrt{g_s} R_s, \quad \Rightarrow \quad \frac{c}{16\pi G_s} \int d^4 x \sqrt{g_4} R_4 L^d. \tag{3.1}$$

We see that in this frame the four dimensional Newton constant is $G_s/(cL^d)$, and so it depends on the expectation value of the four dimensional field L. To suppress this dependence we rescale the metric

$$g_{4\mu\nu} \rightarrow g_{E\mu\nu} = (L/L_0)^d g_{4\mu\nu}, \tag{3.2}$$

where L_0 is a constant. This leads to the standard 4d EH action

$$S_1 = \frac{cL_0^d}{16\pi G_s} \int d^4 x \sqrt{g_E} R_E, \tag{3.3}$$

where the 4D Newton constant does not depend on the expectation value of L

$$G_N = \frac{G_s}{cL_0^d}. \tag{3.4}$$

Now that the kinetic terms of L and the graviton do not mix we can turn to the various static contributions to $V(L)$.

- Contributions due to (3+p)-branes:

  By (3+p)-branes we mean branes that wrap p cycles in M and all the four non-compact directions. Namely, from a four dimensional perspective these are space-filling branes. Their action in the frame (3.1) is

  $$N_{p+3} T_{3+p} \int d^4 x \sqrt{g_4} L^p, \tag{3.5}$$

  where N_{p+3} is the number of (3+p)-branes and T_{3+p} is their tension. To find the potential in the Einstein frame we rescale (3.2). This rescaling gives a factor of $(L_0/L)^{d/2}$ for each of the four non-compact directions. Therefore the potential is

  $$V_{3+p} = N_{3+p} T_{3+p} L_0^{2d} \frac{1}{L^{2d-p}}. \tag{3.6}$$

  It is interesting to note that this potential goes to zero in the decompactification limit ($L \rightarrow \infty$). This is a somewhat counterintuitive result since in this limit the size of the brane blows up like L^p and so does the action (3.5). It is only due to the rescaling to the Einstein frame that the potential vanishes.

- Contributions due to l-fluxes:

  The relevant term in the $4 + d$ dimensional action is

  $$\int d^s x \sqrt{g_s} c_l F_l^2, \tag{3.7}$$

  where the c_l's are constants that typically depend on the value of the other moduli. Now suppose that we have N units of F_l (i.e. $\int_{l-cycle} F_l = N_l$) then we find in the 4d effective action (in the Einstein frame) the following potential for L

  $$V_{l-flux} = N_l^2 c_l L_0^{2d} \frac{1}{L^{d+2l}}. \tag{3.8}$$

- Contributions due to the curvature of M:

  The EH action in $4 + d$ leads also to the following term in 4D

  $$\frac{1}{G_s} \int d^4 x \sqrt{g_4} (L_0/L)^{2d} \left(\int d^d x \sqrt{g_d} R_d \right). \tag{3.9}$$

  On dimensional ground $\int d^d x \sqrt{g_d} R_d = k_M L^{d-2}$ where k_M is a dimensionless parameter that (depending on the topology of M) can be negative zero or positive. Thus we find

  $$V_{cur} = \frac{k_M L_0^{2d}}{G_s} \frac{1}{L^{d+2}}. \tag{3.10}$$

  Note that when M is a CY manifold k_M vanishes.

There are also non-perturbative contributions due to (-1+p)-branes and gaugino condensation. These are exponentially small in L and will not play a role here.

When attempting to use these potentials to construct models of inflation it is important to make sure that the scalar is canonically normalized. This ensures that also in dynamical situations, such as inflation, the potential has the usual interpretation. In the case at hand we denote the canonically normalized scalar field associated with L by ϕ and the two are related in the following way (for $d = 6$)

$$L = e^{\alpha\phi} \quad \text{with} \quad \alpha = \frac{1}{\sqrt{24}}. \tag{3.11}$$

Before discussing IPI let us show that none of the terms discussed above leads to inflation by itself. With a single contribution $V(L)$ takes the form

$$V(\phi) = \frac{V_0}{L^C} = V_0 \exp\left(-\sqrt{\frac{2}{k}}\phi\right) \quad \text{with} \quad k = 48/C^2. \tag{3.12}$$

Such a potential is known to yield exact cosmological solutions [23]. For a flat universe

$$ds^2 = -dt^2 + a(t)^2 dx_i^2 \quad i = 1, 2, 3 \tag{3.13}$$

we have

$$a(t) = a_0 t^k, \tag{3.14}$$

$$\phi = \sqrt{2k} \log \left(\sqrt{\frac{V_0}{k(3k-1)}} t \right),$$

which implies a universe with

$$w = \frac{P}{\rho} = -1 + \frac{2}{3k}, \tag{3.15}$$

where as usual P is the pressure and ρ is the energy density. Since all the examples mentioned above have $C \geq 7$ and since the condition for an accelerating universe is $k > 1$ we see from (3.12) that none of the stringy contributions to $V(L)$ give an accelerating universe, let alone a universe with n_s close to 1.[4] This is believed [24] to be a general result valid for all moduli, not necessarily the radion.

It is therefore particularly interesting that a combination of these potentials can give an inflection point inflation. For this to happen we need the potential to get contributions from three terms

$$V = a_1 \exp(-j_1 \alpha \phi) + a_2 \exp(-j_2 \alpha \phi) + a_3 \exp(-j_3 \alpha \phi), \tag{3.16}$$

with, say, $j_1 > j_2 > j_3$, $a_1, a_3 > 0$ and $a_2 < 0$.

Note that the requirement $a_2 < 0$ can be satisfied since not all the terms discussed above are positive definite. V_{cur}, for example, can be negative. When L is small this might lead to some non-trivial effects [25, 26] due to winding modes that become light. We, however, are interested in the large L case in which nothing dramatic is expected to happen if V_{cur} is negative. Another way to generate a negative potential is via orientifolds. Their tension is negative and so they induce negative V_{3+p}. This puts some mild constraints on the possible values of N_{3+p} that can be satisfied. To conclude, using V_{p+3}, V_{l-flux} and V_{cur} one can construct many examples of inflection point inflation in string theory.

For concreteness in the rest of the paper we focus on a particular example with $j_1 = 12$, $j_2 = 10$ and $j_3 = 8$. We emphasize, however, that the conclusions we present below do *not* depend on this particular choice. In particular, the stringy resolution of the overshoot problem works equally well for other choices of j_1, j_2 and j_3.

For $j_1 = 12$, $j_2 = 10$ and $j_3 = 8$ an inflection point at $\phi = \phi_{\text{inflection}} = \sqrt{24} \log(L_{\text{inflection}})$ is obtained if we take

$$\frac{a_1}{a_3} = \frac{2}{3} L_{\text{inflection}}^4, \qquad \frac{a_2}{a_3} = -\frac{8}{5} L_{\text{inflection}}^2. \tag{3.17}$$

Since an overall rescaling of the potential does not change $\phi_{\text{inflection}}$ only ratios of the a's appear in this condition. From (3.17) we see that, as expected, to have a potential with an inflection point we need to fine tune the parameters in the potential

$$\frac{a_2^2}{a_1 a_3} = \frac{96}{25}. \tag{3.18}$$

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[4]Moreover even if there was a stringy potential with a small enough C to accelerate the universe there would still be the problem of a graceful exit. Namely, with such a potential inflation will not end. This is, of course, interesting for models of quintessence but not for inflation.

In the last section we shall see that this condition can be relaxed.

Expanding near the inflection point we find that

$$V_0 = \frac{a_3}{15 L_{\text{inflection}}^8}, \quad \beta = \frac{a_3 \sqrt{24}}{54 L_{\text{inflection}}^8}, \tag{3.19}$$

where we are using the notation of eq. (2.1), and so the COBE normalization condition, (2.6), gives

$$L_{\text{inflection}} \approx 6.7 \, a_3^{1/8} \sqrt{N}, \tag{3.20}$$

which implies that for typical values of a_3 the supergravity approximation is valid. This is not surprising since usually the COBE normalization condition implies that the energy scale associated with inflation is a few orders of magnitude smaller than the Planck scale.

4. The overshoot problem and its stringy resolution

As explained above the main drawback of models of IPI is that they suffer from the overshoot problem. This problem is generic and appears also in the stringy realization of models of IPI discussed in the previous section. This is illustrated in figure 1. In this figure we see how sensitive inflation is to the initial condition. If the initial condition is near the inflection point then the number of e-foldings, N, is large. But if the initial condition is even slightly away from the inflection point, inflation does not occur since ϕ overshoots $\phi_{\text{inflection}}$. Interestingly enough string theory provides a simple dynamical resolution to this problem which we shall now discuss.

In the previous section we described some of the static terms in $V(L)$. There are, however, also time dependent contributions to $V(L)$. These are due to particles with masses that depend on L. Denoting the particle densities by n_i, the potential they induce is

$$V_i(L, t) = n_i(t) m_i(L). \tag{4.1}$$

This potential is time-dependent because $n_i(t)$ dilutes as the universe expands

$$n_i(t) \sim \frac{1}{a^3(t)}. \tag{4.2}$$

Most known examples in string theory, such as perturbative states, yield $m(L) \to 0$ in the decompactified limit, and so $V_i(L, t)$ vanishes in this limit and does not help much with the overshoot problem. However, non-perturbatively there are (0+p)-branes (namely, branes that wrap p cycles in M and are point-like objects in the four dimensional uncompactified space-time) with $m(L) \to \infty$ in the decompactified limit.

To be precise the mass of a (0+p)-brane is

$$m_{0+p} = T_{0+p} L^p \left(\frac{L_0}{L} \right)^{d/2}, \tag{4.3}$$

where T_{0+p} is the tension of the brane. The factor of L^p is due to the volume of brane, while the $(L_0/L)^{d/2}$ factor is due to the transformation to the Einstein frame (coming from the
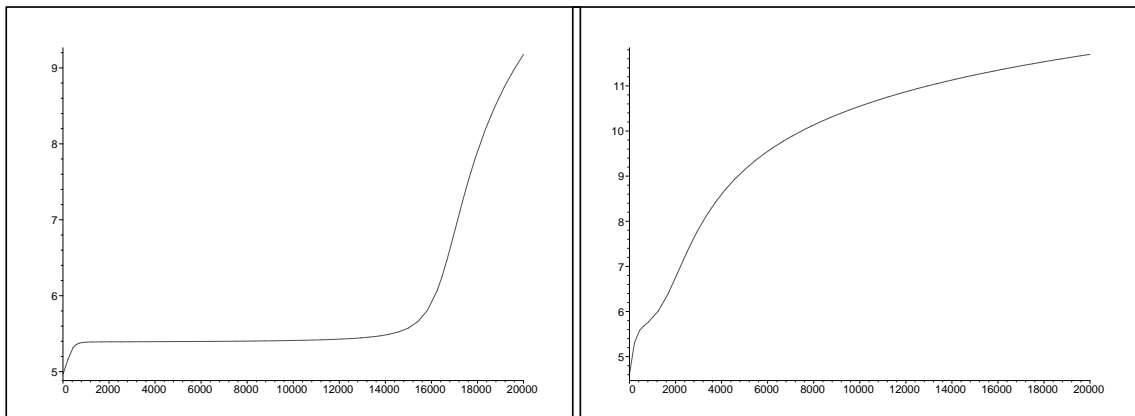
Figure 1: A numerical illustration of the overshoot problem: In both cases we take $L_{\text{inflection}} = 3$ and display ϕ as a function of time. In the picture to the left we take $\phi_{\text{initial}} = \frac{12}{13}\phi_{\text{inflection}}$ and obtain a decent amount of inflation when ϕ crosses $\phi_{\text{inflection}} \approx 5.38$. In the picture to the right we take $\phi_{\text{initial}} = \frac{6}{7}\phi_{\text{inflection}}$. Despite the fact that this is fairly close to $\phi_{\text{inflection}}$ inflation is not generated.

$\sqrt{g_{tt}}$ in the particle action). We see that, unlike in the case of $(3+p)$-branes (3.6), now, for $p > d/2$, the volume term dominates and we find that $m(L) \to \infty$ in the decompactification limit. Therefore, such $(0+p)$-branes lead to time dependent potentials that blow up when $L \to \infty$

$$V_{0+p} = n_{0+p}T_{0+p}L_0^3 L^{p-3}. \tag{4.4}$$

We would like to argue now that this kind of time dependent potential is exactly what is needed to resolve the overshoot problem of IPI. The basic idea is simple[5] and illustrated in figure 2. The static piece of the potential is denoted by the red/solid line and the time dependent piece of the potential by the green/dashed line. In (a),(b) the static potential is steep, but since there has not been significant expansion the time-dependant potential is able to balance it, preventing $\phi(t)$ from acquiring a large velocity. Heuristically, $\phi(t)$ (denoted by the blue/filled circle) follows the dilution of the time dependent potential. In (c) $\phi(t)$ enters the shallow region (where the slow roll parameters are small) where inflation takes place and the time dependent potential starts to slow down exponentially. In (d) $\phi(t)$ is dominated solely by the static potential but its velocity is now low enough to allow prolonged inflation.

To see if this heuristic argument actually holds we have to solve the equations of motion which for a flat FRW universe are (we take the reduced Planck mass to be 1)

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V_{\text{static}} + V_{0+p},$$
$$\dot{n}_{0+p} = -3Hn_{0+p}, \tag{4.5}$$
$$\ddot{\phi} + 3H\dot{\phi} = -\frac{d}{d\phi}\left(V_{\text{static}} + V_{0+p}\right)$$

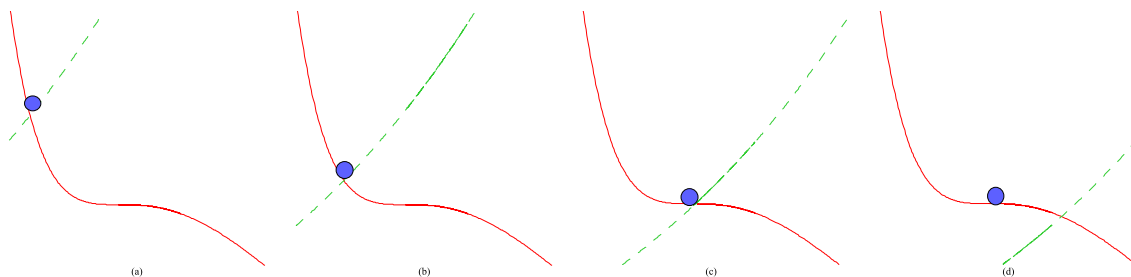[5]In fact, it has some similarities with thermal inflation [27, 28].

Figure 2: A heuristic demonstration of how a time dependent potential, that scales like $1/a^3(t)$, can resolve the overshoot problem of IPI.

$L_{\text{inflection}}$	2	5	8	10	15	20	25
$n_{0+6}^{initial-min}$	0.098877	0.005431	0.00081	0.000316	0.000054	0.000015	0.000005

Table 1: The minimal value of $n_{0+6}^{initial}$ that gives enough inflation for some values of $L_{inflection}$.

where V_{static} is given by (3.16), (3.17) with $j_1 = 12$, $j_2 = 10$, $j_3 = 8$ and V_{0+p} by (4.4). We assume that other moduli fields, in particular the dilaton, are not too extreme and so all the constants that appear in V_{static} and V_{0+p} are of order 1 in string units. We will get back to this point momentarily.

The question that we wish to address is whether a generic initial condition will give a large enough N to be consistent with experiment. Since we are working within the supergravity approximation, the approximation breaks down at $L < 1$. This means that we should take $L_{\text{inflection}} > 1$ (for the approximation to be valid during inflation) and that the most natural initial condition is $L_{\text{initial}} \sim 1$ (which gives $\phi_{\text{initial}} \sim 0$). Recall that the illustration in the beginning of this section shows (for $n_{0+p} = 0$) that for these kind of initial conditions the inflaton overshoots the inflection point and the universe does not inflate. In fact figure 1 shows that this happens already when $L_{\text{initial}}/L_{\text{inflection}} = 6/7$.

To see what happens for $n_{0+p} > 0$ we have to solve (4.5) numerically. On general grounds we expect $n_{0+p}^{initial-min}$, which is the minimal initial value of n_{0+p} (with $\phi_{\text{initial}} = 0$ and $\dot{\phi}_{\text{initial}} = 0$) that is needed to generate a significant amount of inflation, to become smaller as we increase $L_{\text{inflection}}$. The reason is simply that V_{static} becomes weaker at large L while V_{0+p} becomes stronger.

Table 1 summarizes our findings for $p = 6$, $a_3 = 1$ and $N = 100$.

We see that the needed values of n_{0+p}^{initial} are small and become smaller and smaller as we increase $L_{\text{inflection}}$. This is particularly important because it implies that in the supergravity region, $L_{\text{inflection}} \gg 1$, a tiny n_{0+p}^{initial} is sufficient to slow down ϕ enough by the inflection point for the universe to inflate significantly.

The nice feature of this mechanism is that a small n_{0+p}^{initial} is expected to be generated by quantum effects and need not be imposed by hand! The reason is the following. When $L_{\text{initial}} \sim 1$ the Hawking temperature associated with the initial vacuum energy is $T_H \sim \sqrt{V_{\text{initial}}} \sim 1$. Since the mass of the $(0 + p)$-brane is also of order 1 we expect n_{0+p}^{initial} to be of order 1 as well. Thus n_{0+p}^{initial} bigger than the values that appear in the table above is

expected to be generated quantum mechanically.

Note that many other degrees of freedom will be excited at such a high temperature and one might wonder if these could alter our conclusion. We do not expect this to be the case since the effect such excitation could have on the inflanton becomes negligible (compared to the (0+p)-branes) by the time L approaches $L_{\text{inflection}}$. The reason is that the mass of the (0+p)-branes grows with L while the masses of all other excitation does not.

This discussion depends, of course, on the value of the other moduli that we assumed to be stabilized. In particular the value of the dilaton has a significant effect since $T_H \sim g$ while $M_{0+p} \sim 1/g$, and so a natural n_{0+p}^{initial} scales like $e^{-M_{0+p}/T_H} \sim e^{-1/g^2}$. This means that one cannot send the string coupling constant to zero while fixing $L_{\text{inflection}}$. However, as the table above shows, for a large enough $L_{\text{inflection}}$ we can have g that is considerably smaller than 1.

5. Concluding remarks

We end with some comments:

- Our numerical simulations indicate that the mechanism described here works quite generally. For example it works fine also for $3 < p < 6$, though it is not as efficient in the sense that $n_{0+p}^{initial-min}$ is a bit larger than $n_{0+6}^{initial-min}$. As mentioned above it also works for other choices of j_1, j_2 and j_3.

- Here we focused on the case where the radion is the inflaton and show that models of IPI are common in string theory and resolve the overshoot problem. It should be interesting to see if this can be generalized to other setups of IPI. A particularly interesting one is the one of [8 – 10].

- Taking n_{0+6}^{initial} to be bigger than $n_{0+6}^{initial-min}$ by a factor of order 1 leads to N that is practically infinite. This implies that a large enough N is generated with rather generic initial conditions even when the potential does not have an exact inflection point but rather an approximated inflection region. Namely, eq. (3.18) can be relaxed. Thus one needs to tune but not fine tune the parameters in the potentials. This also means that stringy corrections to the potential are unlikely to change our main conclusions. The fact that eq. (3.18) can be relaxed is particularly important since a_1, a_2 and a_3 are quantized once all other moduli are fixed (which is our working assumption).

  To demonstrate this behavior, we explore the range of deviation from the exact inflection point parameters in our example above that would still allow enough e-foldings ($N > 60$) with $n_{0+6}^{\text{initial}} = \frac{1}{100}$. Table 2 shows the maximal deviation for a_1 per value of $L_{\text{inflection}}$, according to $a_1 = a_1 + \delta a_1$ (we allow a_1 to vary in the direction that causes V to lose its extremal point and monotonically approach zero):

$L_{\text{inflection}}$	5	8	10	15	20	25
δa_1	0.01	0.08	0.2	1	3	8

Table 2: The maximal deviation of a_1 that gives enough inflation as a function of $L_{inflection}$.

We see that as we increase $L_{\text{inflection}}$ the sensitivity to the parameters of the potential decreases, and in particular since δa_1 is of order 1 the quantization conditions of the a_i's can be satisfied.

Acknowledgments

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